Forecasting Example Problems with Solutions

1. The Instant Paper Clip Office Supply Company sells and delivers office supplies to companies, schools, and agencies within a 50-mile radius of its warehouse. The office supply business is competitive, and the ability to deliver orders promptly is a big factor in getting new customers and maintaining old ones. (Offices typically order not when they run low on supplies, but when they completely run out. As a result, they need their orders immediately.) The manager of the company wants to be certain that enough drivers and vehicles are available to deliver orders promptly and that they have adequate inventory in stock. Therefore, the manager wants to be able to forecast the demand for deliveries during the next month. From the records of previous orders, management has accumulated the following data for the past 10 months:

Month	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.
Orders	120	90	100	75	110	50	75	130	110	90

a. Compute the monthly demand forecast for February through November using the naive method.

b. Compute the monthly demand forecast for April through November using a 3-month moving average.

c. Compute the monthly demand forecast for June through November using a 5-month moving average.

d. Compute the monthly demand forecast for April through November using a 3-month weighted moving average. Use weights of 0.5, 0.33, and 0.17, with the heavier weights on the more recent months.

e. Compute the mean absolute deviation for June through October for each of the methods used. Which method would you use to forecast demand for November?

Solution:

a. The naive method simply uses the demand for the current month as the forecast for the next month: $F_{t+1} = D_t$. So for February we would have $F_{\text{Feb.}} = D_{\text{Jan.}} = 120$. Similarly, $F_{\text{Nov.}} = D_{\text{Oct.}} = 90$. See the table below for the other months.

b. For a simple 3-month moving average, we take the average of the previous three months' demand as our forecast for next month: $F_{t+1} = \dfrac{D_t + D_{t-1} + D_{t-2}}{3}$. Since we need at least three months to compute the average, and we only have data beginning in January, April is the earliest month for which we can compute the forecast: $F_{\text{Apr.}} = \dfrac{D_{\text{Mar.}} + D_{\text{Feb.}} + D_{\text{Jan.}}}{3} = \dfrac{100 + 90 + 120}{3} = 103.3$. The forecasts for the other months are reported in the table below.

c. The 5-month moving average is similar to the 3-month moving average, except now we take the average of the previous five months' demand. We start with the forecast for June (since we need at least five months' worth of previous demand): $F_{\text{Jun.}} = \dfrac{D_{\text{May}} + D_{\text{Apr.}} + D_{\text{Mar.}} + D_{\text{Feb.}} + D_{\text{Jan.}}}{5} = \dfrac{110 + 75 + 100 + 90 + 120}{5} = 99.0$. The forecasts for the remaining months are computed similarly, and the values are reported in the table below.

d. Simple moving averages (like parts *b* and *c* above) place an equal weight on all of previous months. A weighted moving average allows us to put more weight on the more recent data. For a weighted 3-month moving average we have $F_{t+1} = w_1 D_t + w_2 D_{t-1} + w_3 D_{t-2}$. (Note that the weights should add up to 1.) Using the weights specified in the question, the forecast for April is computed as $F_{\text{Apr.}} = 0.5(D_{\text{Mar.}}) + 0.33(D_{\text{Feb.}}) + 0.17(D_{\text{Jan.}}) = 0.5(100) + 0.33(90) + 0.17(120) = 100.1$. Forecasts for May through November are reported in the table below.

		Forecast			
		Naive	3-Month	5-Month	3-Month
Month	Orders	Method	Moving Avg.	Moving Avg.	Weighted Avg.
Jan.	120	—	—	—	—
Feb.	90	120	—	—	—
Mar.	100	90	—	—	—
Apr.	75	100	103.3	—	100.1
May	110	75	88.3	—	85.8
Jun.	50	110	95.0	99.0	96.8
Jul.	75	50	78.3	85.0	74.1
Aug.	130	75	78.3	82.0	72.7
Sep.	110	130	85.0	88.0	98.3
Oct.	90	110	105.0	95.0	110.7
Nov.	?	90	110.0	91.0	103.4

e. Mean absolute deviation is one measure of how close the forecast is to the actual demand. Recall that forecast error is simply $E_t = D_t - F_t$, and that the absolute deviation is simply the absolute value of error: $|E_t|$. For example, the error for the Naive Method for June is $E_{\text{Jun.}} = D_{\text{Jun.}} - F_{\text{Jun.}} = 50 - 110 = -60$. To compute the mean absolute deviation, take the absolute value of each error term, add them up, and divide by the number of terms: $\text{MAD} = \frac{\sum |E_t|}{n}$. (*Note:* You must take the absolute value of each error term *before* adding them up!) In this case, we compute the mean over five months. The error and MAD for the months June through October are reported below. In general, the forecast accuracy increases as more information is incorporated into the forecast.

		Error ($E_t = D_t - F_t$)			
		Naive	3-Month	5-Month	3-Month
Month	Orders	Method	Moving Avg.	Moving Avg.	Weighted Avg.
Jun.	50	−60	−45.0	−49.0	−46.8
Jul.	75	25	−3.3	−10.0	0.9
Aug.	130	55	51.7	48.0	57.3
Sep.	110	−20	25.0	22.0	11.8
Oct.	90	−20	−15.0	−5.0	−20.7
	MAD	36.0	28.0	26.8	27.5

2. PM Computer Services assembles customized personal computers from generic parts. Formed and operated by part-time UMass Lowell students Paulette Tyler and Maureen Becker, the company has had steady growth since it started. The company assembles computers mostly at night, using part-time students. Paulette and Maureen purchase generic computer parts in volume at a discount from a variety of sources whenever they see a good deal. Thus, they need a good forecast of demand for their computers so that they will know how many parts to purchase and stock. They have compiled demand data for the last 12 months as reported below.

Period	Month	Demand	Period	Month	Demand
1	January	37	7	July	43
2	February	40	8	August	47
3	March	41	9	September	56
4	April	37	10	October	52
5	May	45	11	November	55
6	June	50	12	December	54

a. Use exponential smoothing with smoothing parameter $\alpha = 0.3$ to compute the demand forecast for January (Period 13).

b. Use exponential smoothing with smoothing parameter $\alpha = 0.5$ to compute the demand forecast for January (Period 13).

c. Paulette believes that there is an upward trend in the demand. Use trend-adjusted exponential smoothing with smoothing parameter $\alpha = 0.5$ and trend parameter $\beta = 0.3$ to compute the demand forecast for January (Period 13).

d. Compute the mean squared error for each of the methods used.

Solution:

a. The formula for exponential smoothing is: $F_{t+1} = F_t + \alpha(D_t - F_t)$. To determine the forecast for January, F_{13}, we need to know the forecast for December, F_{12}. This, in turn, requires us to know the forecast for November, F_{11}. So we need to go all the way back to the beginning and compute the forecast for each month. For Period 2, we have $F_2 = F_1 + \alpha(D_1 - F_1)$. But how do we get the forecast for Period 1? There are several ways to approach this, but we'll just use the demand for Period 1 as both *demand* and *forecast* for Period 1. Now we can write $F_2 = F_1 + \alpha(D_1 - F_1) = 37 + 0.3(37 - 37) = 37$. For Period 3 we have $F_3 = F_2 + \alpha(D_2 - F_2) = 37 + 0.3(40 - 37) = 37.9$. The forecasts for the other months are show in the table below. For Period 13 we have $F_{13} = F_{12} + \alpha(D_{12} - F_{12}) = 50.85 + 0.3(54 - 50.85) = 51.79$.

b. For $\alpha = 0.5$ we follow the same exact procedure as we did in part *a.* See the table below for the forecast values.

c. Incorporating a trend simply requires us to include a bit more information. The formula is: $F_{t+1} = A_t + T_t$ where $A_t = \alpha D_t + (1 - \alpha)(A_{t-1} + T_{t-1})$ and $T_t = \beta(A_t - A_{t-1}) + (1 - \beta)T_{t-1}$. Once again we need to go back to the beginning in order to find the necessary values to plug into the formula, and once again we need to make some assumptions about our initial values. For Period 2, we have $F_2 = A_1 + T_1$, so to get the process started, let $A_0 = 37$ and $T_0 = 0$. We can now compute A_1 and T_1 as follows: $A_1 = \alpha D_1 + (1 - \alpha)(A_0 + T_0) = 0.5(37) + (1 - 0.5)(37 + 0) = 37$, and $T_1 = \beta(A_1 - A_0) + (1 - \beta)T_0 = 0.3(37 - 37) + (1 - 0.3)(0) = 0$. Therefore, the forecast for Period 2 is $F_2 = A_1 + T_1 = 37 + 0 = 37$. For Period 3, we first compute A_2 and T_2 as follows: $A_2 = \alpha D_2 + (1 - \alpha)(A_1 + T_1) = 0.5(40) + (1 - 0.5)(37 + 0) = 38.5$, and $T_2 = \beta(A_2 - A_1) + (1 - \beta)T_1 = 0.3(38.5 - 37) + (1 - 0.3)(0) = 0.45$. The forecast for Period 3 is $F_3 = A_2 + T_2 = 38.5 + 0.45 = 38.95$. The forecasts for the remaining months are reported in the table below.

			Expon. Smooth.	Expon. Smooth.	Trend-Adjusted Expon. Smooth. ($\alpha = 0.5$, $\beta = 0.3$)		
Period	Month	Demand	$\alpha = 0.3$	$\alpha = 0.5$	A_t	T_t	F_t
1	Jan.	37	37.00	37.00	37.00	0.00	37.00
2	Feb.	40	37.00	37.00	38.50	0.45	37.00
3	Mar.	41	37.90	38.50	39.98	0.76	38.95
4	Apr.	37	38.83	39.75	38.87	0.20	40.73
5	May	45	38.28	38.38	42.03	1.09	39.06
6	Jun.	50	40.30	41.69	46.56	2.12	43.12
7	Jul.	43	43.21	45.84	45.84	1.27	48.68
8	Aug.	47	43.15	44.42	47.05	1.25	47.11
9	Sep.	56	44.30	45.71	52.15	2.41	48.31
10	Oct.	52	47.81	50.86	53.28	2.02	54.56
11	Nov.	55	49.07	51.43	55.15	1.98	55.30
12	Dec.	54	50.85	53.21	55.56	1.51	57.13
13	Jan.	?	51.79	53.61			57.07

e. To compute the mean square error, first compute the error for each period: $E_t = D_t - F_t$. Take that number and square it, then take the average over all periods: $\text{MSE} = \frac{\sum E_t^2}{n}$. (*Note:* You must square the error terms *before* adding them up!) Take the Exponential Smoothing method with $\alpha = 0.3$, for example. In the month of April, the error is $E_{\text{Apr.}} = D_{\text{Apr.}} - F_{\text{Apr.}} = 37 - 38.83 = -1.83$. We square this value, add it to the other squared error terms, and divide by 12 to get the mean. The error, squared error, and MSE for each of the methods are reported below. The trend-adjusted forecast, which incorporates the most information, has the highest accuracy (lowest MSE).

		Expon. Smooth. $\alpha = 0.3$		Expon. Smooth. $\alpha = 0.5$		Trend-Adj. $\alpha = 0.5, \beta = 0.3$	
Month	Demand	E_t	E_t^2	E_t	E_t^2	E_t	E_t^2
Jan.	37	0.00	0.00	0.00	0.00	0.00	0.00
Feb.	40	3.00	9.00	3.00	9.00	3.00	9.00
Mar.	41	3.10	9.61	2.50	6.25	2.05	4.20
Apr.	37	−1.83	3.35	−2.75	7.56	−3.73	13.93
May	45	6.72	45.14	6.63	43.89	5.94	35.24
Jun.	50	9.70	94.15	8.31	69.10	6.88	47.33
Jul.	43	−0.21	0.04	−2.84	8.09	−5.68	32.26
Aug.	47	3.85	14.86	2.58	6.65	−0.11	0.01
Sep.	56	11.70	136.85	10.29	105.86	7.69	59.20
Oct.	52	4.19	17.55	1.14	1.31	−2.56	6.55
Nov.	55	5.93	35.19	3.57	12.76	−0.30	0.09
Dec.	54	3.15	9.94	0.79	0.62	−3.13	9.78
		MSE	31.31		22.59		18.13

Forecasting Formulas

Simple Moving Average

$$F_{t+1} = \frac{D_t + D_{t-1} + D_{t-2} + \cdots + D_{t-n+1}}{n}$$

Weighted Moving Average

$$F_{t+1} = w_1 D_t + w_2 D_{t-1} + \cdots + w_n D_{t-n+1}$$

Exponential Smoothing

$$\begin{aligned} F_{t+1} &= \alpha D_t + (1-\alpha) F_t \\ &= F_t + \alpha (D_t - F_t) \end{aligned}$$

Trend-Adjusted Exponential Smoothing

$$F_{t+1} = A_t + T_t$$

where $A_t = \alpha D_t + (1-\alpha)(A_{t-1} + T_{t-1})$

and $T_t = \beta (A_t - A_{t-1}) + (1-\beta) T_{t-1}$

Error

$$E_t = D_t - F_t$$

Mean Squared Error

$$\text{MSE} = \frac{\sum E_t^2}{n}$$

Mean Absolute Deviation

$$\text{MAD} = \frac{\sum |E_t|}{n}$$